King Fahd University of Petroleum and Minerals
College of Computer Sciences and Engineering
Information and Computer Science Department

ICS 254: Discrete Structures II
Fall semester 2016-2017 (161)
Major Exam #1, Wednesday October 19, 2016
Time: **120** Minutes

Name: _____

ID#: _____

Instructions:
1. The exam consists of 8 pages, including this page, containing 6 questions.
2. Answer all questions. **Show all the steps.**
3. Make sure your answers are **clear** and **readable**.
4. The exam is closed book and closed notes. **No calculators** or any helping aides are allowed. Make sure you turn off your mobile phone and keep it in your pocket.
5. If there is no space on the front of the page, use the back of the page.

Question	Maximum Points	Earned Points
1	25	
2	20	
3	15	
4	15	
5	15	
6	10	
Total	**100**	

Q1: **[25 points] Evaluate the following.**

a) [3 points] $-105 \bmod 19$

b) [6 points] $(33^3 \bmod 17)^2 \bmod 19$

c) [8 points] $(CA)_{16} = (\qquad\qquad\qquad\qquad)_5$

d) [8 points] $(4831)_9 \times (121)_9$

Q2: [20 points] Solve the following questions

a) [6 points] Prove that if n is an odd positive integer, then $n^2 \equiv 1 \pmod 8$.

b) [6 points] Given an integer represented in base 16 (hexadecimal), find a rule to determine whether it is divisible by 5.

c) [8 points] Using the modular exponentiation algorithm, find $31^{10} \pmod{14}$

Q3: [15 points]

a) [6 points] If p is prime, what are the possible values of the $lcm(45,5p)$?

b) [9 points] Let $n = 12^3$. How many divisors does n have, including 1 and n itself?

Q4: [15 points]

a) [8 points] Find gcd(4611, 3538) using the Euclidean algorithm.

b) [7 points] Express the greatest common divisor of 4611 and 3538 as a linear combination of these two numbers.

Q5: [15 points]

a) [6 points] Find the solutions of the congruence $15x^2 + 19x \equiv 5 \pmod{11}$. [*Hint:* The above congruence is equivalent to the congruence $15x^2 + 19x + 6 \equiv 0 \pmod{11}$.]

b) [9 points] Solve the following system of linear congruences

$$3x \equiv 5 \pmod{8}$$
$$4x \equiv 7 \pmod{9}$$
$$7x \equiv 1 \pmod{11}$$

Q6: [10 points] Using Fermat's little theorem, compute $6^{1066} \bmod 29$